1. (a) Find the PLU factorization (*PA* = *LU*) of the matrix

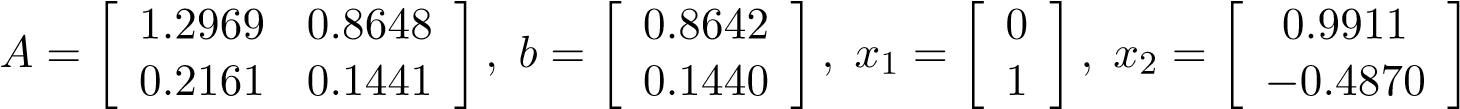
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Description automatically generated with medium confidence

using Gaussian elimination with partial pivoting.

* 1. Use the PLU factorization and forward and back substitution to solve the linear system *Ax* =*b*, with *b* = [1*,*0*,*1*,*0]*t*.

1. Consider the matrix, right side vector, and two approximate solutions,

 *.*

1. Show that *x* = [2*,*−2]*t* is the exact solution of *Ax* = *b*.
2. Compute the error and residual vectors for *x*1 and *x*2.
3. Find ||*A*||∞*,*||*A*−1||∞, and the condition number *κ*∞(*A*).
4. Consider the linear system *Ax* = *b* where A black background with a black square

   Description automatically generated with medium confidence *.* The true answer is seen to be *x* = [1*,* 1]*T*.
5. Determine *TJ* and *TGS*, the Jacobi and Gauss-Seidel iteration matrices, respectively.
6. Find the ∞ − norm and spectral radius of *TJ* and *TGS*.
7. Perform 5 iterations of both the Jacobi and Gauss-Seidel methods using *x*(0) = [0*,*0]*T*. For each present the results in a table with the following format:

column 1: *k* (iteration step)

column 2: A black background with a black square

Description automatically generated with medium confidence(1st component of computed solution vector at step *k*)

column 3: A black background with a black square

Description automatically generated with medium confidence(2nd component of computed solution vector at step *k*)

column 4: k*e*(*k*)k∞ (error norm at step *k*)

column 5: k*e*(*k*)k∞*/*k*e*(*k*−1)k∞ (ratio of successive error norms at step *k*)

Which method is converging fastest and why?

1. Consider the linear system

2*x*1 − *x*2 + *x*3 = −1*,*

2*x*1 + 2*x*2 + 2*x*3 = 4*,*

−*x*1 − *x*2 + 2*x*3 = −5

The system has the exact solution (1*,*2*,*−1)*T* .

1. Is the coefficient matrix of the above system diagonally dominant?
2. Determine *TJ* and *TGS*, the Jacobi and Gauss-Seidel iteration matrices, respectively.
3. Find the ∞ − norm and spectral radius of *TJ* and *TGS*.
4. What can you say about the convergence of the problem with respect to Jacobi and Gauss seidel iteration?
5. Solve the system numerically using Jacobi’s Method, Gauss–Seidel. In each case, let *x* iteration after step *k* if ||*x*(*k*) − *x*(*k*−1)||∞ *<* 10−7. You may modify the python code provided or write your own. Report your results as in problem 3. State clearly your observations of the performance of two methods on this problem.

AFTER THIS ASSIGNMENT YOU SHOULD KNOW THE FOLLOWING

1. Gauss elimination: Elimination Matrices, Elementary matrices ,LU decomposition PA=LU decomposition,

2.Compute the norm of a: matrix, vector, residual vector, error vector.

3. Jacobi, Gauss Siedel: Iteration matrices, spectral radius, diagonally dominant matrices conditions for convergence.